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Traversable Wormholes

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Abstract: We give a brief introduction to some concepts related to traversable wormholes, and review some recent work in the field. This article is part of the course project for 231C, General Relativity, Spring 2019 at UCSB. You can also find this on my website http://web.physics.ucsb.edu/~diandian/.

Contents

1 Introduction 1

2 Terminologies 1

3 Classical constraints 3

4 Quantum constraints 4

5 Realization of traversable wormholes 5

6 Creation of traversable wormholes 8

1 Introduction

Traversable wormholes are great objects as they stimulated a large number of science fiction writers to write about time travels and fast-than-light journeys, which in turn fascinates young students and draws them to research in General Relativity.

There are many words and phrases closely related to traversable wormholes, such as multi-verses, black holes, and grandfathers, but how are they related exactly? In this short note, we give some definitions of several concepts involved, state some conditions to rule certain things out, and give some hopefully illuminating examples.

We will avoid unnecessary equations and try to explain things using more physically intuitive words. We will also focus on the inter-connections between different concepts, examples, and theorems rather than the detailed equations, setups, or proofs involved with each of them. If interested in the details, the reader is directed to the references. Throughout the text, we will restrict our discussion to Lorentzian wormholes, i.e. wormholes in Lorentzian spacetimes.

2 Terminologies

Roughly speaking, a *wormhole* is a handle or a tunnel that connects two different regions of spacetime. If the two regions are in the same universe, it is called an *intra-universe* wormhole, while if the two regions are in two different universes, it is called an *inter-universe* wormhole. However, a wormhole does not need to have only two ends, and the internal structure can be more complicated. So more generally, if a spatial slice has a trivial boundary topology and a non-trivial interior topology, we say there is an intra-universe wormhole. For an inter-universe wormhole, it turns out that topology constraints are not sufficient to charaterise it [1]. In this case, on a certain spatial slice, we manually identify a throat to be a wormhole and call it an inter-universe wormhole. For example, Figure

1 shows a baby universe connected to its parent. The space is topologically trivial, but geometrical reasons sometimes make it more natural to interpret the throat as a wormhole instead of just some curved region of space.

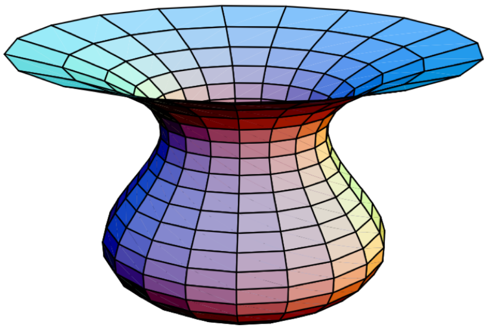

Figure 1. A baby universe connected to its parent. Here it is natural to call the region in the middle a wormhole, although there is no topological non-triviality. Figure taken from [2].

An important feature of a wormhole is its traversability. Roughly speaking, if an entity can enter from one side of the wormhole and exit from the other, the wormhole is traversable. If this cannot be done, but two entities entering from both sides can nevertheless meet somewhere within the tunnel, the tunnel is a non-traversable wormhole. This is the case for a two-sided black hole. (We can also consider the time-reversed situation of this non-traversable wormhole, where a white hole replaces the black hole. In this case, the two universes are still connected in a similar sense, but we do not call it a wormhole for its inability to take things in.) A more rigorous definition for a traversable wormhole exists: if a spacetime contains a causal path that begins and ends at spatial infinity and cannot be continuously deformed to a causal curve that lies entirely in the spatial asymptotic region, the spacetime contains a *traversable* wormhole [3].

If a wormhole is traversable, there is a pivotal feature that divides them into two families: its length. Roughly speaking, if it takes shorter to go through the wormhole than to go around, we say the wormhole is *short*. On the other hand, if it takes longer to go through it than to go around, it is *long*. Since this is only defined for traversable wormholes, traversability is implied when these words are used. Notice that the definition implies that inter-universe traversable wormholes are short, as the trip from one universe to the other without going through the wormhole is impossible, i.e. infinitely long.

As traversable wormholes are frequently associated with time travels, we should define this concept. Although travelling to the far future is technically also time travelling, by far the most popular concept involved here is to do with time travel paradoxes such as the grandfather paradox. To restrict attention to these pathological events, we define the concept of a time machine: If a spacetime contains a closed timelike curve, then the spacetime contains a *chronology-violating time machine*, and this curve traverses the time machine. If we replace the word "timelike" with "causal", we get the definition of a *causality-violating time machine*. We will not distinguish the two concepts and call both of them *time machines* because of the following theorem [1].

Theorem 1. If a smooth spacetime contains a causality-violating time machine that is not a chronology-violating time machine, then there exist infinitesimal perturbations of the metric that result in a new spacetime that contains a chronology-violating time machine.

Certain energy conditions will also be useful for the discussion. In particular, we define Averaged Null Energy Condition (ANEC) to be

$$\int_{-\infty}^{+\infty} T_{\mu\nu} k^\mu k^\nu d\lambda \geq 0 \tag{2.1}$$

for all complete null geodesics, where k^μ is the tangent vector with affine parameter λ.

Finally, we also want to talk about wormhole creations. Since intra-universe wormholes have non-trivial topologies, wormhole creation within a single universe means a topology-changing process. Given a spacetime that admits a time-function, if there is no topology change as a function of time and a wormhole exists on every constant-time hypersurface, we say that the wormhole is *eternal*. It is important to note that an eternal wormhole might only be traversable for a certain amount of time, and a change of traversability does not require a change of topology. If an eternal wormhole is traversable for all time, we say it is an *eternally traversable* wormhole.

3 Classical constraints

Classically, wormholes exist. The maximally extended Schwarzschild is an example. This is a non-traversable wormhole, as nothing from one asymptotic region can reach the other. Things from two universes can only meet inside the black hole.

Traversable wormholes do not exist if ANEC is satisfied. This is a result of the topological censorship.

Theorem 2. In a globally hyperbolic asymptotically flat spacetime satisfying ANEC, every causal curve from $\mathscr{I}^-$ to $\mathscr{I}^+$ is homotopic to a topologically trivial curve from $\mathscr{I}^-$ to $\mathscr{I}^+$, where $\mathscr{I}^+$ and $\mathscr{I}^-$ are future and past null infinity respectively. (Friedman, Schleich, Witt [4])

By Raychaudhuri's equation, violation of ANEC means that light rays that focus when entering a wormhole can defocus when getting out [5]. This gives an intuitive way to see why traversable wormholes cannot exist if ANEC is respected. It is expected that Null Energy Condition (and thus ANEC) holds for classical matter, so the topological censorship rules out traversable wormholes in spacetimes coupled with only classical matter.

Classically, causality prevents topology change. This is a result of the following theorem.

Theorem 3. In a Lorentzian spacetime, if there is a timelike tube connecting spacelike surfaces of different topology, then the interior of the timelike tube contains closed timelike curves. (Hawking [6])

To summarize, things are not very exciting classically. If there aren't any wormholes in our universe to start with, we cannot produce any. Even if there are, they are necessarily non-traversable.

4 Quantum constraints

Going to a semi-classical regime, where quantum matter is allowed on a curved spacetime, violation of ANEC is no longer a dream. In fact, there have been explicit constructions of traversable wormholes when ANEC is violated. See [5, 7, 8] for traversable wormholes in asymptotically AdS spacetimes (constructed using ideas from holography), and [3, 9] for asymptotically flat ones. We will review some of these constructions in Section 5. Quantum mechanics also makes wormhole creation possible, at least in the path integral formulism of quantum gravity. An explicit construction is given in [10]. We will review this construction in Section 6.

However, even quantum matter obeys rules, and there are indeed conditions that we believe to hold even in quantum/semi-classical situations. First of all, let us point out a potentially ill existence, following the argument in [11]. Suppose we have a short wormhole and we want to see what we can do with it. For simplicity, let us take the throat to have zero length and small radius so that we effectively have two worldlines identified. Now we boost one end of the wormhole either by gravitational or electrostatic means for some time and then boost it back (see Figure 2). This procedure produces a time-shift between the two ends of the wormhole, just like the twin paradox. If the time delay is sufficiently large compared to the distance between the two ends of the wormhole in the ambient spacetime, we have created a time machine. To see this, imagine a timelike observer going from the point labelled by 9 on the left end of the wormhole in Figure 2 and following a timelike curve to reach the other point labelled by 9. Since the two points are identified, this observer would have moved along a closed timelike curve. Therefore, by definition, a time machine is created. This story tells us that short wormholes can lead to causality violation, and they should not be allowed, even in quantum situations.

A condition that rules this out is the Self-Consistent Achronal Averaged Null Energy Condition (SCAANEC). SCAANEC states that there is no self-consistent solution in semi-classical gravity in which ANEC is violated on a complete, achronal null geodesic [12]. Achronal means no two points are timelike separated. To get a feeling of how this rules out short wormholes, consider two points A and B, each located near one of the two wormhole ends. Now consider two null curves from A to B, one being the fastest route by going through the wormhole and the other being the fastest route by going around. One of them is achronal because one of the is the fastest route between A and B (otherwise there exists a timelike curve connecting two points on this null curve, so a faster route can be found). If the wormhole is short, the one going through it would be achronal. Now it is both achronal and ANEC is violated along it (since it is a traversable wormhole), SCAANEC says it cannot exist. So intra-universe short wormholes are forbidden. The above construction does not work for inter-universe wormholes as one cannot use the ambient space to form a

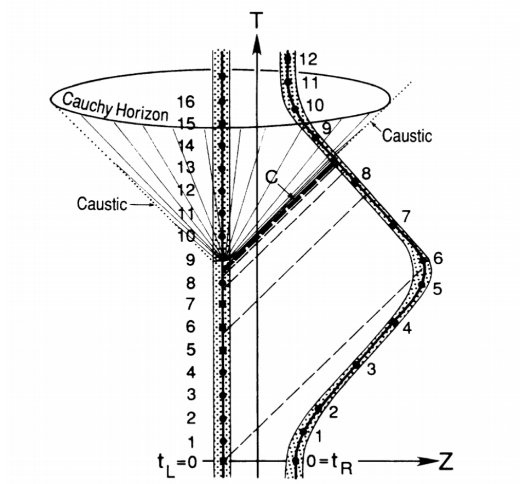

Figure 2. Conversion of a short-throat wormhole to a time machine by having one end of the wormhole boosted. Points labelled by same numbers are identified. Figure taken from [11].

closed timelike curve. However, SCAANEC also rules these out, i.e. it forbids any short wormholes, either inter- or intra-universe.

Do we expect SCAANEC to hold? There are direct evidence and proofs in certain limited scenarios (see e.g. [13, 14]), but a different reason to believe it is that it follows from Generalized Second Law (GSL) [15], a generalization of the ordinary second law of thermodynamics. We will not discuss the issue further, and the reader is directed to [16] for "ten proofs of GSL".

To summarize, quantum mechanics provides much richer physics in the context of wormholes. Traversable wormholes are now allowed. However, SCAANEC requires that short wormholes still do not exist, so it will always take longer to go through the wormhole than to go around. Wormhole creation also becomes possible, but of course, they are necessarily long ones.

Now that we know short wormholes are forbidden by SCAANEC, we can actually go back to the classical case and use this fact to understand why long wormholes are also forbidden there [9]. Equations in classical physics are local, so we can go to the covering space without changing the physics. In the covering space, the wormhole now joins two asymptotic regions instead of one, so any intra-universe wormhole is effectively inter-universe, which is by definition short and prohibited.

5 Realization of traversable wormholes

The very first example of a traversable wormhole construction was given in [5] by Gao, Jefferis and Wall. The key ingredient in this set up is a direct interaction turned on for a finite amount of time between the two Conformal Field Theories (CFTs) living on the boundaries of an eternal BTZ black hole bulk spacetime (See Figure 3). Here the interaction opens up a gap between E_1 and E_2 as shown in the figure, which allows a particle on one side to reach the other side in finite time through the bulk. It has to start early enough so the wormhole does not close up before it reaches it. Therefore, it is an eternal wormhole, but not an eternally traversable one. Key to the construction is the negative energy induced

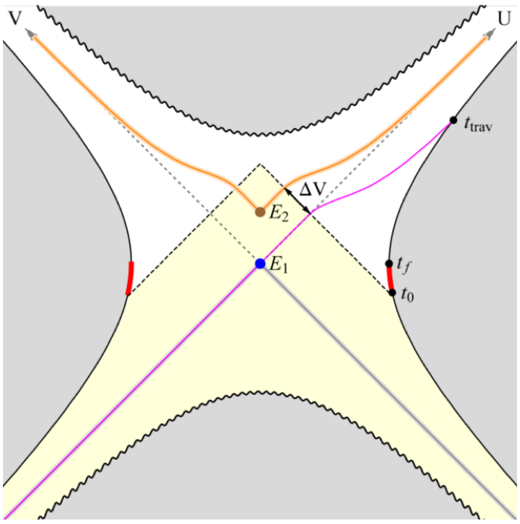

Figure 3. The Gao-Jefferis-Wall traversable wormhole construction by direct boundary coupling. Interaction is turned on between t_0 and t_f on the boundary (labelled red). Part of the event horizon is moved upwards, opening up a gap which is the wormhole. Figure taken from [5].

by the interaction. The direct interaction between two boundary field theories alters the causal structure, so we can no longer say that the wormhole connects two different universes. Thus, the wormhole here is an intra-universe one, although it was clearly inter-universe if the interaction is not turned on. As SCAANEC forbids inter-universe traversable wormholes, starting from an inter-universe non-traversable wormhole, the change of causal structure here is actually necessary.

A stable and eternally traversable wormhole was constructed by Maldacena and Qi in [8]. There, coupling that is not present in the bulk is still needed, but it is time-independent. The construction uses Nearly-AdS_2 gravity, where all gravitational degrees of freedom live on the boundary. For an introduction to Nearly-AdS_2 gravity, see [17]. Features of the Gao-Jefferis-Wall construction was investigated using Nearly-AdS_2 gravity in [7].

In [9] by Maldacena, Milekhin and Popov, time-independent, four-dimensional, asymptotically flat traversable wormholes were found, and no extra boundary interaction was needed. See Figure 4. Three regions labelled by different colours are solved separately and matched at the overlaps. The wormhole region is approximated by $AdS_2 \times S^2$ with some deformation, so the Nearly-AdS_2 techniques mentioned above can be helpful. In this construction charged massless fermion fields are used to provide a Casimir-like energy around a non-contractable loop that threads the wormhole. Note that Casimir energy is non-local; so we cannot go to the covering space and use SCAANEC to forbid it.

In a paper by Fu, Grado-White and Marolf [3], it was shown that certain wormholes generically become traversable after backreaction due to some linear quantum fields. No

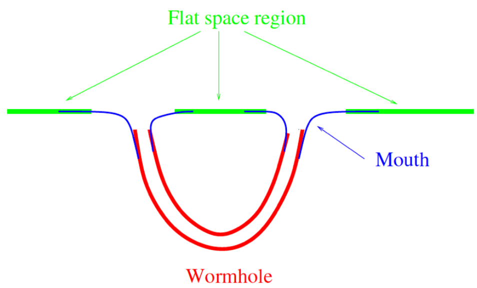

Figure 4. Traversable wormhole construction in asymptotically flat 4D spacetime using fermion fields. The whole spacetime is separated into three regions and equations are solved in each region. The solutions have overlapping regions of validity and are matched there. Figure taken from [9].

additional boundary interaction is needed, and it works for both asymptotically flat and asymptotically AdS spacetimes. For this construction to work, we want backgrounds that

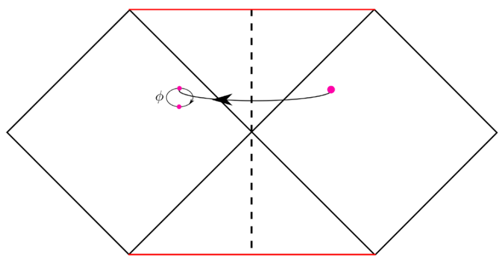

Figure 5. The Fu-Grado-White-Marolf traversable wormhole construction using $\mathbb{Z}_2$ identifications of spacetimes with bifurcate Killing horizons. The identification loses a global time-translation Killing symmetry. Figure taken from [3].

are smooth, globally hyperbolic $\mathbb{Z}_2$ quotients of spacetimes with bifurcate Killing horizons and well-defined Hartle-Hawking states under an isometry that exchanges left- and right-moving horizons. See Figure 5 for how the points are identified: a reflection across the dashed line and a simultaneous antipodal map on the suppressed sphere. We call the original spacetime $\tilde{\mathcal{M}}$ and the quotient spacetime $\mathcal{M}$. We will then put free quantum fields $\tilde{\phi}(\tilde{x})$ on the spacetime $\tilde{\mathcal{M}}$. Fields on $\mathcal{M}$ are then defined by

$$\phi(x)_{\pm} = \frac{1}{\sqrt{2}}[\tilde{\phi}(\tilde{x}) \pm \tilde{\phi}(J\tilde{x})], \tag{5.1}$$

where J denotes the quotient action and the plus and minus signs correspond to periodic and antiperiodic conditions around the homotopy cycle, a resulting feature of the quotient procedure. We are allowed to do this because free fields on $\mathcal{M}$ have a symmetry under $\phi \to -\phi$. The next steps are the following:

- Choose Hartle-Hawking state on $\tilde{\mathcal{M}}$;

- Use (5.1) to define the corresponding Hartle-Hawking states (with two choices of sign) on $\mathcal{M}$;
- Define $T_{ab\pm}(x)$ in terms of the field on $\mathcal{M}$;
- Pull-back $T_{ab\pm}(x)$ to $\tilde{\mathcal{M}}$ (only for ease of calculation), and note that the pulled-back $T_{ab\pm}(\tilde{x})$ is not the same as $\tilde{T}_{ab}(\tilde{x})$ calculated directly using HH state on $\tilde{\mathcal{M}}$;
- Evaluate $T_{kk\pm}$ for the corresponding HH state on $\mathcal{M}$ and show that one of the choices of sign makes it negative;
- Show that violation of ANEC implies traversability when there is enough symmetry and thus a traversable wormhole is found.

Explicit calculations were used to show that ANEC is violated, but the argument is quite general. Perturbative calculations show that (in one of the explicit examples) the wormhole remains traversable for longer and longer as the zero-temperature limit is approached, suggesting that a non-perturbative treatment would give an eternally traversable wormhole.

To summarize, different methods have been used to construct wormholes. Quantum fields are used to violate ANEC. These fields are physical, and physical fields are believed to obey SCAANEC. Therefore, all the examples are long (and thus intra-universe) wormholes.

6 Creation of traversable wormholes

To describe wormhole creation, a theory that allows topology change is needed. Here we use the path integral formulation of quantum gravity and assume that the integral contains a sum over all topologies. In this framework, we can make use of a tool called instantons. *Instantons* are classical solutions to Euclidean field equations with non-zero action. In other words, an instanton is a saddle point of the path integral but not the dominating solution. Instantons that describe creation of wormholes have been found (see e.g. [18]), but the mouths of these wormholes accelerate apart. The Unruh temperature associated with this acceleration makes the wormhole created in an excited state defined by the Unruh temperature. Even if the ground state is traversable, the large number of non-traversable states relative to the traversable ones plus the tiny gap separating the energies of traversable and collapsing wormholes mean that it is not very likely for the wormhole to be found in a traversable state. Therefore, it seems that the acceleration is undesired here, and indeed, if the mouths remain static, the Fu-Grado-White-Marolf construction discussed above immediately suggest traversability.

To counterbalance this acceleration (which provided the energy for wormhole creation in the first place), the construction in [10] used asymptotically AdS backgrounds. The curvature of the background provides a force that drives geodesics closer, and we can tune this "curvature force" to make the created wormhole arbitrarily static. Both spherically symmetric non-vacuum and non-spherically-symmetric vacuum solutions were found, and no exotic matter is used in the background spacetime. It is not clear whether the same can be achieved in an asymptotically flat spacetime without exotic matter in the background, as the construction relied largely on the "AdS property" of the background.

To summarize, the path integral formulation of quantum gravity allows topology change, and if one is willing to tune and wait, traversable wormholes can be created in an asymptotically AdS spacetime.

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